

BRANE-UNIVERSE IN SIX DIMENSIONS

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Growing non-singular solution of 6-dimensional Einstein equations for the 4-brane in infinite transversal 2-space is found. This solution provides gravitational trapping of matter and 4-dimensional gravity on the brane without extra delta-like source. The suitable solution exists in the case of the (2+4)-space and not exists for the (1+5)-signature.

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After the papers [1, 2, 3] the models with brane-Universe in a high dimensional space-time have become popular. Large extra dimensions may play an important role in solving such problems as smallness of cosmological constant, the origin of the hierarchy, the nature of flavor, the source of SUSY breaking, etc. As in previous articles [3, 4, 5, 6, 7], in the present paper the Universe is considered as a single brane.

Brane models need some natural matter localization mechanism and explanation of 4-dimensional Newton's law. The question of matter localization on the brane has been investigated in various papers (for recent ideas see [8]). In our opinion the localizing force must be universal for all types of 4-dimensional matter fields. In our world gravity is known to be the unique interaction which has universal coupling with all matter fields. If extended extra dimensions exist, it is natural to assume that trapping of matter on the brane has a gravitational nature. Because of problems with quantum field theory in curve space it is difficult to find exact trapping mechanism for different matter fields separately. Our approach is more general: to provide universal and stable trapping we assume that on the brane (where all gravitating matter can reside) gravitational potential should have minimal value with respect to extra coordinates.

Growing gravitational potential is the opposite choice compared to the one of Randall-Sundrum with the maximum on the brane [2]. We assume that 4-dimensional Newton's law on the brane is the result of the cancellation mechanism introduced in [3, 6] (as it is in Randall-Sundrum's case also), which allows both types of gravitational potential. This mechanism can be used even for a simple 5-dimensional model with exponentially growing transversal gravitational potential. Far from the brane (natural cutoff is the width of the brane) transversal gravitational potential becomes large and usual splitting of 5-dimensional space of [2, 3, 4, 5] cannot be valid any more. This is similar to the case of QCD where nobody worries about the increasing quark potential.

In this paper we introduce a realistic model of gravity and matter localization on (1+3)-brane embedded in a (2+4)-space in the case of growing transversal gravitational potential. Our motivation for the choice of the signature is as follows. In the massless field case (weakest coupling with gravity) symmetries of a multi-dimensional manifold can be restored. It is well known, that in the zero-mass limit the main equations of physics are invariant under the 15-parameter nonlinear conformal transformations. A long time ago it was also discovered that the conformal group can be written as a linear Lorentz-type transformation in a (2+4)-space (for these subjects see for example [9]).

The (2+4)-space was earlier investigated in [7], but for a (2+3)-brane with two open time directions. The case of (1+3)-brane in six dimensions had been considered by different authors [10, 11, 12, 13, 14]. In all these papers space-like extra dimensions were investigated. We will show that a suitable solution of our minimal model with growing gravitational potential does not exist for the case of (1+5)-space.

In all (1+5)-models the authors used polar coordinates for the transversal 2-space. At the origin, where the brane is placed, polar coordinates are singular and the choice of acceptable boundary conditions is problematic. In our approach we consider transversal (1+1)-space and use nonsingular Cartesian coordinates.

Einstein's equations in six dimension with a bulk cosmological constant Λ and stress-energy tensor T_{AB} can be written in the form

$${}^6R_{AB} = -\frac{1}{2}\Lambda g_{AB} + \frac{1}{M^4} \left(T_{AB} - \frac{1}{4}g_{AB}T \right), \quad (1)$$

where ${}^6R_{AB}$ and M are respectively the Ricci tensor and the fundamental scale. Capital Latin indices run over $A, B, \dots = 0, 1, 2, 3, 5, 6$.

We are looking for a solution of (1) in the form

$$ds^2 = \phi^2(z)\eta_{\alpha\beta}(x^\nu)dx^\alpha dx^\beta + g_{ij}(z)dx^i dx^j, \quad (2)$$

providing stable splitting of the sub-manifold [5]. Greek indices $\alpha, \beta, \dots = 0, 1, 2, 3$ numerate coordinates in 4-dimensions, while small Latin indices $i, j, \dots = 5, 6$ - coordinates of the transversal space.

In (2) only the 4-dimensional conformal factor ϕ^2 and the metric tensor of transversal (1+1)-space g_{ij} depends on the extra coordinates x^i via the dimensionless coordinate

$$z = \frac{x_5^2 - x_6^2}{\epsilon^2} \geq 0, \quad (3)$$

where ϵ is the width of the brane.

Suppose that the system of Einstein and matter field equations have the solution with the minimal energy when extra coordinates enter stress-energy from the metric (2) only [5]. This means that strength of gauge fields towards the extra directions and covariant derivations of matter fields with respect to extra coordinates are zero $F_{iA}^a = D_i\psi^a = 0$ [5]. Then multi-dimensional matter energy-momentum tensor can be written in the form

$$T_{\alpha\beta} = \frac{\tau_{\alpha\beta}(x^\nu)}{\rho^2\phi^2(z)}, \quad T_{ij} = -g_{ij}(z)\frac{L(x^\nu)}{\rho^2\phi^4(z)}. \quad (4)$$

The Lagrangian of matter fields $L(x^\nu)$ and the 4-dimensional stress-energy $\tau_{\alpha\beta}(x^\nu)$ in the space (2) automatically appears to be independent of z (see for example [9]).

In (4) ρ is an arbitrary length scale. The only length scale of our task is the width of the brane and it is natural to take

$$\epsilon^2 = \rho^2. \quad (5)$$

To localize the matter on the brane without extra sources the factor $1/\phi^2(z)$ in (4) should have a delta-like behavior. It means that $\phi^2(z)$ (and transversal gravitational potential) must be a growing function starting from the brane location. Let us place our world at the origin $z = 0$, corresponding to a (1+3)-brane moving with the speed of light in the transversal (1+1)-space.

Using the metric ansatz (2) one can derive the following decomposition of 6-dimensional Einstein's equations

$$\begin{aligned} R_{\alpha\beta} - \frac{1}{2}\eta_{\alpha\beta} \left(D_i D^i \phi^2 + \frac{1}{2\phi^2} D_i \phi^2 D^i \phi^2 \right) &= -\frac{\Lambda}{2}\phi^2\eta_{\alpha\beta} + \frac{1}{\epsilon^2 M^4 \phi^2} \left(\tau_{\alpha\beta} - \frac{\tau - 2L}{4}\eta_{\alpha\beta} \right), \\ R_{ij} - \frac{2}{\phi^2} \left(D_i D_j \phi^2 - \frac{1}{2\phi^2} D_i \phi^2 D_j \phi^2 \right) &= g_{ij} \left(-\frac{\Lambda}{2} - \frac{\tau + 2L}{4\epsilon^2 M^4 \phi^4} \right), \end{aligned} \quad (6)$$

where R_{ij} and D_i are respectively the Ricci tensor and the covariant derivative containing the metric tensor g_{ij} of the 2-space. The Ricci tensor in four dimensions $R_{\alpha\beta}$ is constructed by the 4-dimensional metric tensor $\eta_{\alpha\beta}(x^\nu)$ in the standard way.

Since the gravity in the 2-space is trivial, we have only one independent component of g_{ij} . We choose

$$g_{ij} = \eta_{ij}g(z), \quad (7)$$

where η_{ij} is the metric tensor of the flat (1+1)-space.

On the brane we require to have 4-dimensional Einstein equations without a cosmological term

$$R_{\alpha\beta} = \frac{1}{\epsilon^2 M^4 \phi^2} \left(\tau_{\alpha\beta} - \frac{1}{2} \eta_{\alpha\beta} \tau \right). \quad (8)$$

Then (6) takes the form

$$\begin{aligned} z(\phi\phi'' + 3\phi'^2) + \phi\phi' &= \left(\phi^2 \Lambda - \frac{\tau + 2L}{2\epsilon^2 M^4 \phi^2} \right) \frac{\epsilon^2}{8} g, \\ \frac{\phi''}{\phi} &= \frac{g'}{g}, \\ z \left(\frac{g''}{g} - \frac{g'^2}{g^2} + \frac{4\phi'}{\phi} \frac{g'}{g} \right) + \frac{g'}{g} + \frac{4\phi'}{\phi} &= \left(\phi^2 \Lambda + \frac{\tau + 2L}{2\epsilon^2 M^4 \phi^2} \right) \frac{\epsilon^2}{4\phi^2} g, \end{aligned} \quad (9)$$

where a prime denotes a derivative with respect to z . We note that the system (9) is self-consistent only for strictly correlated extra coordinates in the form of (3), corresponding to the light-like (1+3)-brane in (1+1)-space.

The solution of the second equation of the system (9) (5 - 6 component of Einstein equations) is

$$g = c\phi', \quad (10)$$

where c is an integration constant.

Because of our choice of the source (4) the last equation of (9) is satisfied identically and only the first equation of the system remains to be solved. The first integral of the remaining equation can be written in the form

$$z\phi^3\phi' + A\phi^5 + B\phi + C = 0, \quad (11)$$

where C is the second integration constant and the dimensionless parameters

$$A = -\frac{\Lambda\epsilon^2 c}{40}, \quad B = \frac{c(\tau + 2L)}{16M^4}, \quad (12)$$

were introduced. In general B depend on the 4-coordinates x^ν .

Using (2) and (10), integration of (8) by the extra coordinates gives the relation

$$m_P^2 = \frac{\int \sqrt{-g} dx_5 dx_6}{\int \frac{1}{\epsilon^2 M^4 \phi^2} \sqrt{-g} dx_5 dx_6} = \epsilon^2 M^4 \frac{\int \phi^4 d\phi}{\int \phi^2 d\phi} \quad (13)$$

between Planck's scale m_P and the fundamental scale M . To have the finite-volume integrals in (13) in the infinite transversal 2-space we need the solution of (11) which approaches some finite value at infinity. For simplicity, in this paper we consider only positive ϕ , since it enters the metric tensor quadratically. In order to avoid singularity the function ϕ and its first derivative $\phi' > 0$, which, according to (10) enters in the metric of transversal space, must be finite and nonzero on the brane.

In the case of pure gravitational field, when $B = 0$, there is no continuous solution of (11), which runs from the origin to $z \rightarrow \infty$. At the point $z = 1$ (corresponding, according to (3), to the brane-width equal to ϵ) $\phi(z)$ becomes infinitely large and in fact the transversal 2-space is closed. The brane has the horizon of the size ϵ towards the extra dimensions, similar to the interior of a black hole. So, 4-dimensional gravity with ordinary Einstein equations (8) is trapped on the brane itself by multi-dimensional gravitation. This is a nonlinear effect of the gravitational field.

When matter sources are added situation changes and there exists a nonsingular solution of (11) growing from the origin to some finite value at the infinity. It means that 4-dimensional matter, which enters (11) via the parameter B , besides acting on the 4-dimensional curvature, also changes the shape of the gravitational potential in the transversal space.

Boundary conditions are taken in the form

$$\phi(z \rightarrow 0) \approx 1 + \frac{z}{|c|}, \quad \phi(z \rightarrow \infty) \approx a - \frac{1}{b|c|z^b} \quad (14)$$

where $a > 1$ is the value of ϕ at the infinity and $b > 0$. This choice corresponds to the following geometry of the bulk space-time on the brane and in the transversal infinity:

$$\begin{aligned} ds^2(z \rightarrow 0) &\approx \eta_{\alpha\beta}(x^\nu) dx^\alpha dx^\beta + \eta_{ij} dx^i dx^j, \\ ds^2(z \rightarrow \infty) &\approx a^2 \eta_{\alpha\beta}(x^\nu) dx^\alpha dx^\beta + \frac{1}{z^{b+1}} \eta_{ij} dx^i dx^j. \end{aligned} \quad (15)$$

At the origin ϕ is assumed to be equal to 1, since any other integration constant in (14) will correspond to an overall rescaling of the coordinates x^A in (15).

Conditions (14) impose certain relations

$$\begin{aligned} Aa^5 + Ba + C &\approx 0, & A + B + C &\approx 0, \\ ba^3 - 5Aa^4 - B &\approx 0, & 1 + 5A + B &\approx 0. \end{aligned} \quad (16)$$

From these relations one can find

$$\begin{aligned} b &\approx \frac{4a^3 + 3a^2 + 2a + 1}{a^3(a^3 + 2a^2 + 3a + 4)}, \\ A &= -\frac{\Lambda\epsilon^2 c}{40} \approx \frac{1}{a^4 + a^3 + a^2 + a - 4}, \\ B &= \frac{c(\tau + 2L)}{16M^4} \approx -\frac{a^4 + a^3 + a^2 + a + 1}{a^4 + a^3 + a^2 + a - 4}. \end{aligned} \quad (17)$$

For the realistic (similar to [1]) values of our physical parameters

$$m_P^2 \gg M^4 \epsilon^2, \quad (\tau + 2L) \sim M^4 > 0, \quad (18)$$

from the relations (13) and (17) follows

$$\begin{aligned} a &\gg 1, & c &\sim -10, & \Lambda &> 0, \\ b &\sim \frac{1}{a^3}, & \epsilon^2 &\sim \frac{1}{\Lambda a^4}, & m_P^2 &\sim M^4 \epsilon^2 a^2. \end{aligned} \quad (19)$$

So, smallness of the 4-dimensional gravitational constant $\sim 1/m_P^2$ and of the width of our world $\sim \epsilon$ can be the result of a large values of the transversal gravitational potential a and bulk cosmological constant Λ .

Since c is negative and ϕ' is positive, as it is seen from (10), a suitable solution of our model does not exist in the case of space-like transversal 2-space studied in [10, 11, 12, 13, 14]. In the present paper we do not consider a pure time-like 2-space.

Using the relations (16) one can show that the solution of (11) has an inflection point on the brane $z = 0$ (at the inflection point second derivative of a function is zero, while the first is not). It means that the transversal curvature R_{zz} is zero on the brane, at the minimum of the transversal gravitational potential. The function ϕ has no other inflection point outside the brane and smoothly grows from 1 to its maximal value a .

To summarize, in this paper it is shown that for the realistic values of the fundamental scale and brane stress-energy, there exists a non-singular static solution of (2+4)-dimensional Einstein equations corresponding to infinite transversal (1+1)-space. This solution provides gravitational trapping of the 4-dimensional gravity and the matter on the brane without extra sources. In contrast to Randall-Sundrum's case, the factor responsible for this trapping is the growing function away from the brane, but has a convergent volume integral, although the transversal 2-space is infinite. In our model any point-like particle in four dimensions can be a (1+1)-dimensional object and there is possibility for nontrivial applications of the classical theory of the strings.

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